ELLIPSE

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 C

$$\frac{2a}{e} = 3(2ae)$$

$$\Rightarrow e^2 = \frac{1}{3} \qquad \Rightarrow e = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow e = \frac{1}{\sqrt{3}}, \left(e = -\frac{1}{\sqrt{3}} \text{ reject}\right)$$

Sol.2 A

$$e = \frac{5}{8} \;;\; 2ae = 10 \quad \Rightarrow 2a = \frac{10}{e} \quad \Rightarrow 2a = 16$$

$$\text{Latus rectum} = \frac{2b^2}{a} = \frac{2a^2(1-e^2)}{a}$$

$$= 2a\,(1 - e^2) = 16\left(1-\frac{26}{64}\right) = \frac{39}{4}$$

Sol.3 A

$x = 3\,(\cos t + \sin t) \qquad y = 4\,(\cos t - \sin t)$

$$\Rightarrow \frac{x}{3} = \cos t + \sin t \;; \quad \frac{y}{4} = \cos t - \sin t$$

square & add $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 2$

Ellipse Equation $\dfrac{x^2}{18} + \dfrac{y^2}{32} = 1$

Sol.4 B

Let P $(a\cos\theta, b\sin\theta)$

OP = 2

$\Rightarrow OP^2 = 4$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta = 4$

$\Rightarrow 6\cos^2\theta + 2\sin^2\theta = 4$

$$\cos\theta = \pm\frac{1}{\sqrt{2}} \quad \Rightarrow \theta = \pm\frac{\pi}{4}$$

Sol.5 C

$F_1(3, 3)$; $F_2(-4, 4)$

$2ae = F_1F_2$

$$2ae = \sqrt{(3+4)^2 + (3-4)^2}$$

$$2ae = 5\sqrt{2} \qquad \text{....(1)}$$

mid point of P_1P_2 will be centre of ellipse

centre $\left(-\dfrac{1}{2}, \dfrac{7}{2}\right)$

$$\text{Ellipse } \frac{\left(x+\frac{1}{2}\right)^2}{a^2} + \frac{\left(y-\frac{7}{2}\right)^2}{b^2} = 1$$

Passing through origin $\dfrac{1}{4a^2} + \dfrac{49}{4b^2} = 1$ ...(2)

From (1) and (2) $e = \dfrac{5}{7}$

Sol.6 B

$\dfrac{x^2}{18} + \dfrac{y^2}{32} = 1 \quad a < b$

Tangent Equation slope form

$x = my + \sqrt{a^2m^2 + b^2}$

$$\text{Slope} = \frac{1}{m} = -\frac{4}{3} \;\Rightarrow\; m = -\frac{3}{4}$$

$$x = -\frac{3}{4}y + \sqrt{32\left(\frac{9}{16}\right) + 18}$$

$4x + 3y = 24$

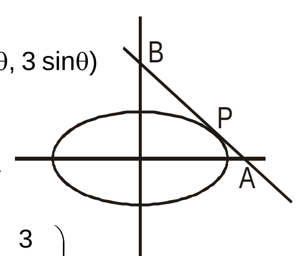
$\dfrac{x}{6} + \dfrac{y}{8} = 1$

Intercept on axis is 6 and 8

So area of Δ CAB = $\dfrac{1}{2} \times 6 \times 8 = 24$ sq. units.

Sol.7 A

$\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$



Let the point P$(4\cos\theta, 3\sin\theta)$

Tangent at P

$$\frac{x}{4}\cos\theta + \frac{y}{3}\sin\theta = 1$$

$$A\left(\frac{4}{\cos\theta}, 0\right) \;;\; B\left(0, \frac{3}{\sin\theta}\right)$$

Let the middle point M(h, k)

$2h = \dfrac{4}{\cos\theta} \Rightarrow \cos\theta = \dfrac{2}{h}$

$2k = \dfrac{3}{\sin\theta} \Rightarrow \sin\theta = \dfrac{3}{2k}$

square & add

$\dfrac{4}{h^2} + \dfrac{9}{4k^2} = 1$

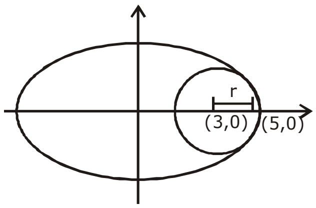
$16k^2 + 9h^2 = 4h^2k^2$

$16y^2 + 9x^2 = 4x^2y^2$

Sol.8 B

$2a = 10 \Rightarrow a = 5$; $2b = 8 \Rightarrow b = 4$

$e = \sqrt{1 - \dfrac{b^2}{a^2}} = 3/5$



Focus $(\pm ae, 0) \Rightarrow (\pm 3, 0)$

$r = 5 - 3 = 2$

$\Rightarrow r = 2$

Sol.9 B

$y = mx \pm \sqrt{(a^2+b^2)m^2 + b^2}$ ....(1)

$y = mx \pm \sqrt{a^2m^2 + (a^2+b^2)}$ .....(2)

Eq^n (1) and (2) are same

$(a^2 + b^2)\, m^2 + b^2 = a^2\, m^2 + a^2 + b^2$

$m^2 = a^2/b^2 \Rightarrow m = \pm\, a/b$

$\Rightarrow by = ax \pm \sqrt{a^4 + b^4 + a^2b^2}$

Sol.10 D

$\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$

Tangent

$x = my \pm \sqrt{a^2m^2 + b^2}$

$x = my \pm \sqrt{16m^2 + 25}$

$(x - my) = \pm\sqrt{16m^2 + 25}$

Passes through (4, 5)

$(4 - 5m)^2 = 16m^2 + 25$

$16 + 25m^2 - 40m - 16m^2 - 25 = 0$

$9m^2 - 40\,m - 9 = 0 \;\begin{cases} \nearrow m_1 \\ \searrow m_2 \end{cases}$

$m_1 + m_2 = \dfrac{40}{9}$; $m_1m_2 = -1$

Angle between tangents will be $\dfrac{\pi}{2}$

Sol.11 D

$\dfrac{de}{dt} = 0.1$ $\qquad e^2 = 1 - \dfrac{b^2}{a^2} = 1 - \dfrac{3}{4}$

$e = 0.1\,t + c$ $\qquad \Rightarrow e = \dfrac{1}{2}$

when $t = 0$, $e = \dfrac{1}{2}$

$\Rightarrow c = 0.5$

$e = 0.1\,t + 0.5$

ellipse become auxiliary circle where $e \to 1$

$1 = 0.1\,t + 0.5 \Rightarrow t = 5$ sec.

Sol.12 C

tangent at P :

$\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ .....(1)

tangent at Q $(a\cos\theta, a\sin\theta)$

$x\cos\theta + y\sin\theta = a$ .....(2)

solving (1) & (2)

$y = 0$

Sol.13 A

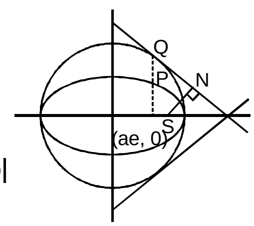
$P(a\cos\alpha, b\sin\alpha)$

$Q\,(a\cos\alpha, a\sin\alpha)$

Tangent at Q point

$x\cos\alpha + y\sin\alpha = a$

$SN = |ae(\cos\alpha - a)|$



$SP = \sqrt{(ae - a\cos\alpha)^2 + b^2\sin^2\alpha}$

$= \sqrt{a^2e^2 + a^2\cos^2\alpha - 2a^2e\cos\alpha + b^2 - b^2\cos^2\alpha}$

$= \sqrt{a^2 + \cos^2\alpha(a^2 - b^2) - 2a^2e\cos\alpha}$

$= |ae\cos\alpha - a|$

$\Rightarrow SP = SN$

Sol.14 A

Same as Ques. no. (13)

Ans.(A) Isosceles triangle

Sol.15 B

Positive end of L.R. $P\left(ae, \dfrac{b^2}{a}\right)$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$$

$$y' = -\frac{x_1 b^2}{y_1 a^2} \;;\; m_N = \left.\frac{y_1 a^2}{x_1 b^2}\right|_P = \frac{1}{e}$$

Equation of normal

$$y - \frac{b^2}{a} = \frac{1}{e}(x - ae)$$

$$ey - \frac{eb^2}{a} = x - ae$$

$$ey - \frac{ea^2(1-e^2)}{a} = x - ae$$

$ey - ae + e^3a = x - ae$

$x - ey - e^3a = 0$

Sol.16 B

$ax\sec\theta - by\,\mathrm{cosec}\theta = a^2 - b^2$

$$\text{slope} = \frac{a\sec\theta}{b\,\mathrm{cosec}\,\theta} = \frac{5}{3}$$

$$\frac{\sec\theta}{\mathrm{cosec}\,\theta} = 1$$

$$\tan\theta = 1 \;\Rightarrow\; \theta = \frac{\pi}{4}$$

Sol.17 C

$P(a\cos\theta, b\sin\theta)$

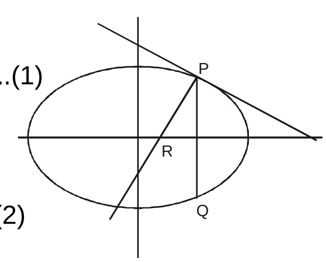
Normal at P ; $ax\sec\theta - by\,\mathrm{cosec}\theta = a^2 - b^2$

$$R\left(\frac{a^2-b^2}{a\sec\theta}, 0\right)$$

Let mid point of PR is M(h, k)

$$2h = \frac{a^2-b^2}{a\sec\theta} + a\cos\theta$$

$$\cos\theta = \frac{2ha}{2a^2-b^2} \quad ....(1)$$



$2k = b\sin\theta$

$$\Rightarrow \sin\theta = \frac{2k}{b} \quad ....(2)$$

Square & odd

$$\frac{4h^2a^2}{(2a^2-b^2)^2} + \frac{4k^2}{b^2} = 1$$

$$\frac{4a^2x^2}{(2a^2-b^2)^2} + \frac{4y^2}{b^2} = 1 \text{ Ellipse}$$

Sol.18 B

$$\frac{x^2}{10} + \frac{y^2}{4} = 1$$

$T = S_1$

$$\frac{xx_1}{10} + \frac{yy_1}{4} = \frac{x_1^2}{10} + \frac{y_1^2}{4}$$

passes through (2, 1)

$$\frac{2x}{10} + \frac{y}{4} = \frac{4}{10} + \frac{1}{4}$$

$4x + 5y = 13$

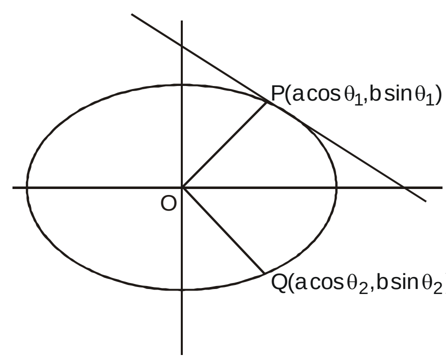
Sol.19 B

By using property

$(S_1F_1) . (S_2F_2) = b^2 = 3$

Sol.20 B

$$M_{OP} = \frac{b\sin\theta_1}{a\cos\theta_1} = \frac{b}{a}\tan\theta_1$$



$$\tan\theta_1\tan\theta_2 = -\frac{a^2}{b^2}$$

$$M_{OQ} = \frac{b}{a}\tan\theta_2$$

$$M_{OP} \times M_{OQ} = \frac{b^2}{a^2}\tan\theta_1\tan\theta_2$$

$$= \left(\frac{b^2}{a^2}\right)\left(\frac{-a^2}{b^2}\right) = -1$$

So right angle at centre.